SMT, Strings, Security

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Plan

- String constraints by example
- A word equation primer
- Decidable fragments of string constraints

Strings in Verification

String in verification

```
// Pre = (true)
String s= '';
// P1 = (s ∈ ε)
while(*){
    // P2 = (s = u · v ∧ u ∈ a* ∧ v ∈ b* ∧ |u| = |v|)
    s= 'a' + s + 'b';
}
// P3 = P2
assert(!s.contains('ba') && (s.length() % 2) == 0);
// Post = P3
```

String in verification

ASCII, Unicode

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}
// P3 = P2
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// Post = P3
```

String in verification

Regular expression
assertion: $s = \epsilon$

// $Pre = (\quad)$

```
String s='';
```

// $P_1 = (s \in \epsilon)$

```
while(*){
```

// $P_2 = (s = u \cdot v \wedge u \in a^* \wedge v \in b^* \wedge |u| = |v|)$

```
    s= 'a' + s + 'b';
}
```

// $P_3 = P_2$

```
assert(!s.contains('ba') && (s.length() % 2) == 0);
```

// $Post = P_3$

String in verification

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// Pre = (true)
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      s= 'a' + s + 'b';
}
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assert(!s.contains('ba') && (s.length() % 2) == 0);
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```

Word/string **concatenation**

String in verification

Loop invariant combining **word equations**, **regex** constraints, **length** constraints

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// P3 = P2
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```

Substring constraint

String in verification

```
// Pre = (true)
String s= '';
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while(*){
    // P_2 = (s = u · v ∧ u ∈ a* ∧ v ∈ b* ∧ |u| = |v|)
    s= 'a' + s + 'b';
}
// P_3 = P_2
assert(!s.contains('ba') && (s.length() % 2) == 0);
// Post = P_3
```

Or **regex:**

$s \notin \Sigma^* \cdot ba \cdot \Sigma^*$

String in verification

```
// Pre = (true)
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while(*){
    // P2 = (s = u · v ∧ u ∈ a* ∧ v ∈ b* ∧ |u| = |v|)
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assert(!s.contains('ba') && (s.length() % 2) == 0);
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```

Presburger length constraint

String in verification

```
// Pre = (true)
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     s= 'a' + s + 'b';
}
// P3 = P2
assert(!s.contains('ba') && (s.length() % 2) == 0);
// Post = P3
```

→ Need a solver that supports all those operators!

Alphabets

- All constraints are formulated w.r.t. to some fixed **finite alphabet** Σ
- $\Sigma = \{a, b, c, d\}$
- $\Sigma = \{0, \ldots, 255\}$ (e.g., 8-bit ASCII)
- $\Sigma = \{0, \ldots, 2^{32} - 1\}$ (e.g., UTF-32)

Semantics and notation

- Finite sequences of letters: Σ^*
- Empty word: ϵ
- Concatenation: $x \cdot y$

- Equations: $s = t$
- Regular expressions: $x \in \mathcal{L}$
- Word length: $|x|$

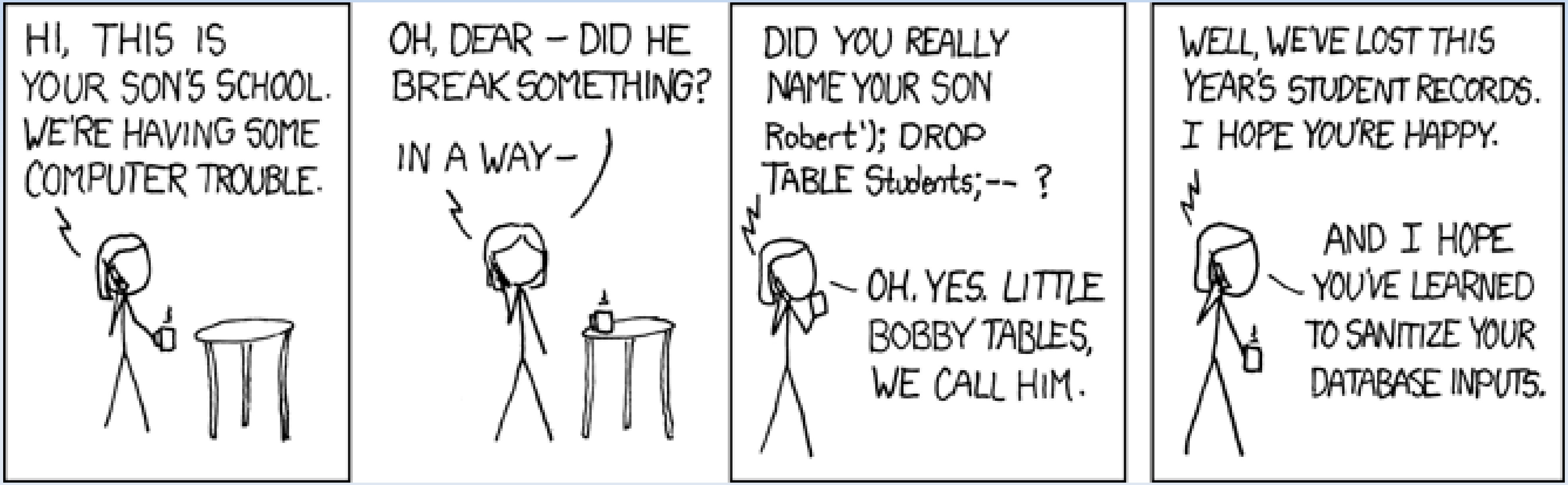
LARGE Alphabets

- Naive use of finite-state automata quickly becomes impossible
  - **Concrete** letters as transition guards → far too many transitions are needed to express interesting languages
  - **Symbolic** handling of letters is necessary

- Sometimes complex string conversion functions necessary, e.g.

  UTF-8 ↔ UTF-32

Injection attacks



xkcd.com

What is happening here?

Possible SQL command in a program

```
database.execute(
    "INSERT INTO students (name) VALUES ('"
    + name
    + "');");
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Command with input substituted

```
INSERT INTO students (name) VALUES ('Robert'); DROP TABLE students;--');
```

What is happening here?

Possible SQL command in a program

```
database.execute(
    "INSERT INTO students (name) VALUES ('"
    + name
    + "');");
```

Command with input substituted

```
INSERT INTO students (name) VALUES ('Robert'); DROP TABLE students;--');
```

Problem:
Input string
ends quotation!

Command
embedded in
user input
is executed

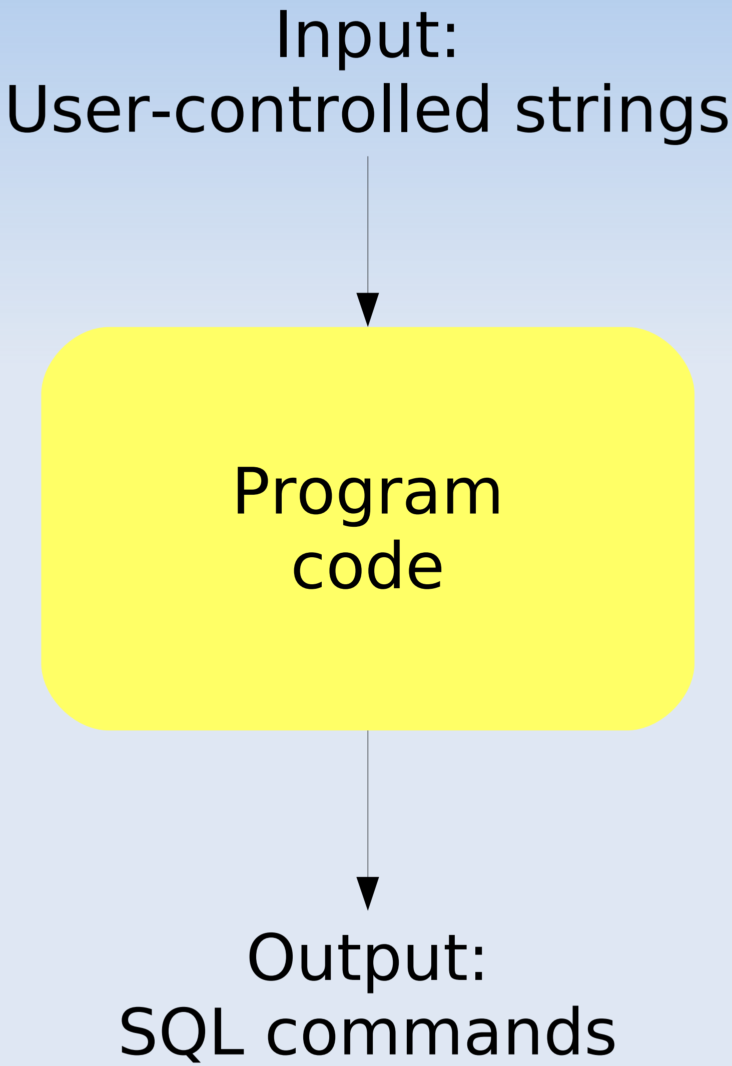
What is happening here?

Possible SQL command in a program

```
database.execute(
    "INSERT INTO students (name) VALUES ('"
    + name
    + "');");
```

- Since no **sanitisation** is applied, program is vulnerable to SQL injection attacks!

How can this be detected?



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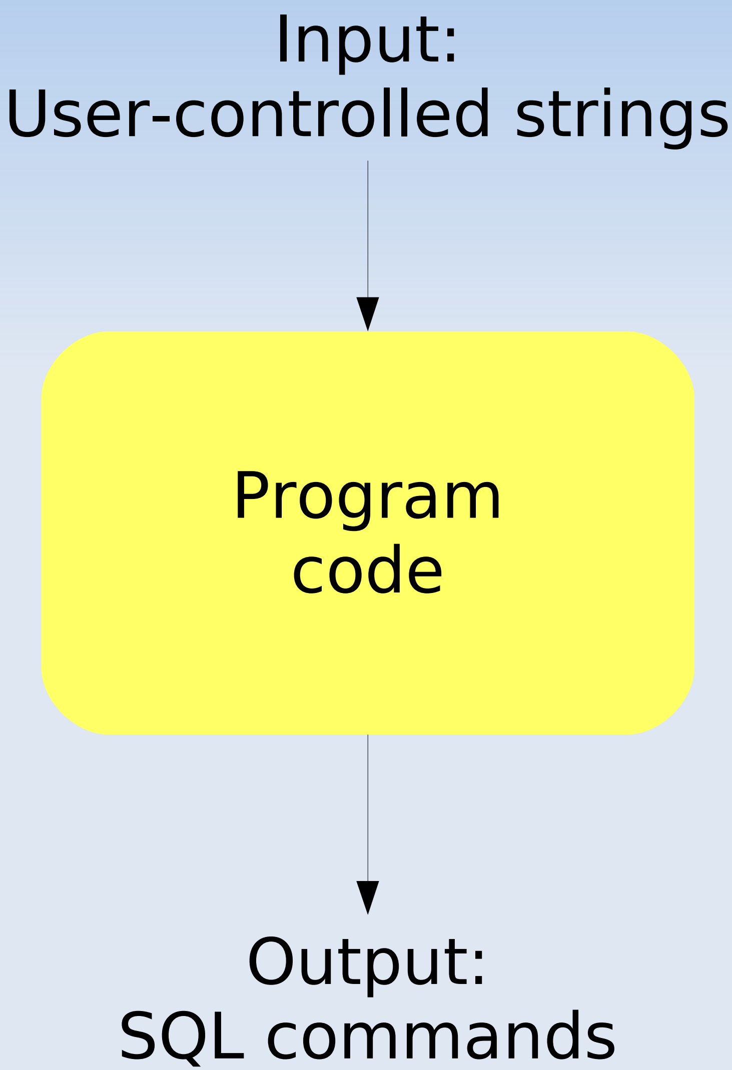

$$name \in \mathcal{L}_{input}$$

$$\wedge$$

$$\phi_{path}$$

$$\wedge$$

$$cmd \in \mathcal{L}_{attack}$$

How can this be detected?

Input:
User-controlled strings

↓

Program
code

↓

Output:
SQL commands

$$name \in \mathcal{L}_{input}$$

$$\wedge$$

$$\phi_{path}$$

$$\wedge$$

$$cmd \in \mathcal{L}_{attack}$$

Regex
or CFG

What is happening here?

Possible SQL command in a program

```
database.execute(
    "INSERT INTO students (name) VALUES ('"
    + name
    + "');");
```

- However, this case could more easily be found with techniques like **taint tracking**
- But what if sanitisation were actually applied?

A subtle XSS vulnerability

JavaScript embedded in a web-page

```
var x = goog.string.htmlEscape(cat);
var y = goog.string.escapeString(x);

catElem.innerHTML =
  '<button onclick="createCatList(\'' +
  y + '\')">' + x + '</button>';
```

A subtle XSS vulnerability

Input string

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A subtle XSS vulnerability

HTML escape: & → &

Input string

JavaScript escape: ' → \'

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A subtle XSS vulnerability

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& → &

Input string

JavaScript escape:
' → \'

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```

Implicit HTML **un**escape
of the `onclick`
attribute:
& → &

An XSS vulnerability (2)

JavaScript embedded in a web-page

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catElem.innerHTML =
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```

One possible attack

Choose `cat` to be `');alert(1);//`

Generated HTML string is then:

```
<button onclick="createCatList('&#39;);alert(1);//')">
&#39;);alert(1);//</button>
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```

Vulnerability since escape functions are applied in **wrong order**

One possible attack

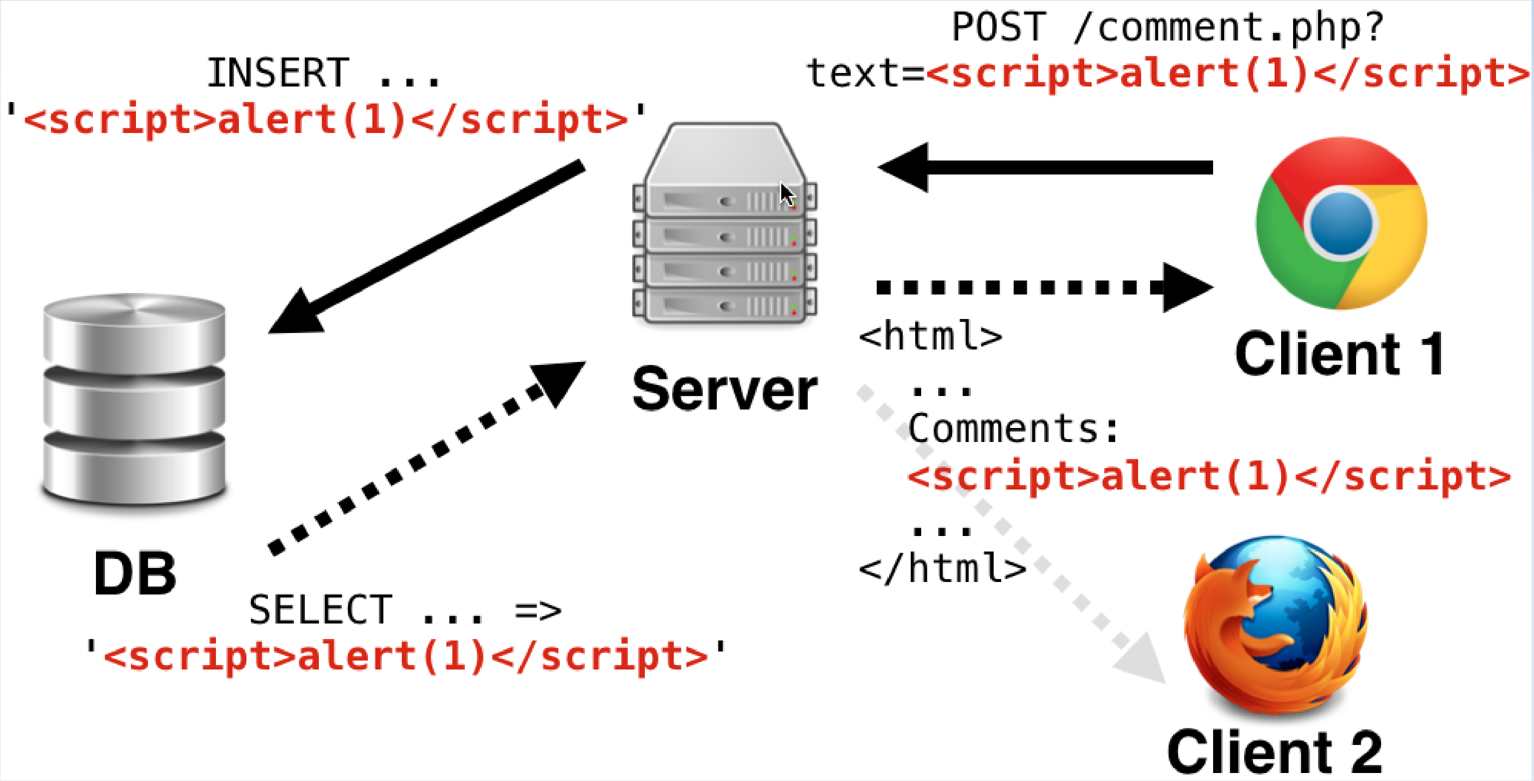
Choose `cat` to be `');`

This will be **unescaped** to `createCatList('');alert(1);//')`

Generated HTML string is then:

```
<button onclick="createCatList('&#39;);alert(1);//')">
&#39;);alert(1);//</button>
```

Cross-site scripting



http://blog.aboutme.vn/choi-xss-tai-knock-xss-moe/

Solvers for escape ops?

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- We need transducers!
  → Automata with multiple tracks

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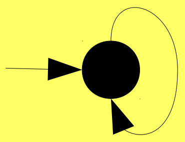
toUpperCase

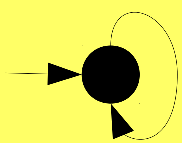


a/A
b/B
c/C
...

Solvers for escape ops?

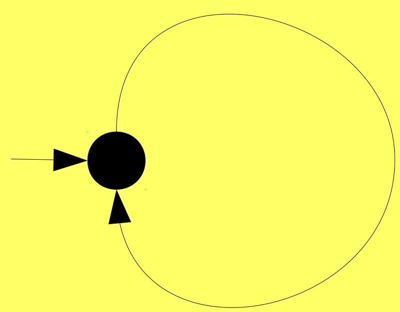
- We need transducers!
  → Automata with multiple tracks

toUpperCase



```
a/A
b/B
c/C
...
```

htmlEscape



```
</&lt;
>/&gt;
&/&amp;
...
```

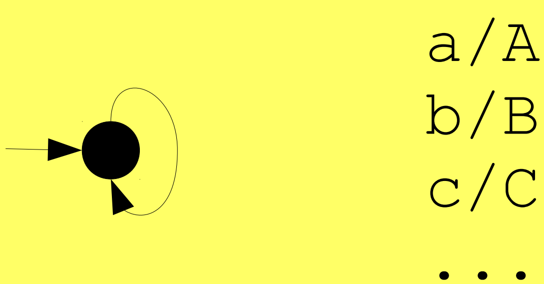
replaceAll

...

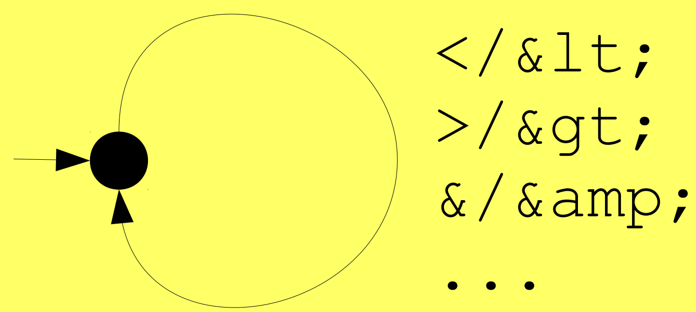
Solvers for escape ops?

- We need transducers!
  → Automata with multiple tracks

toUpperCase



htmlEscape



replaceAll

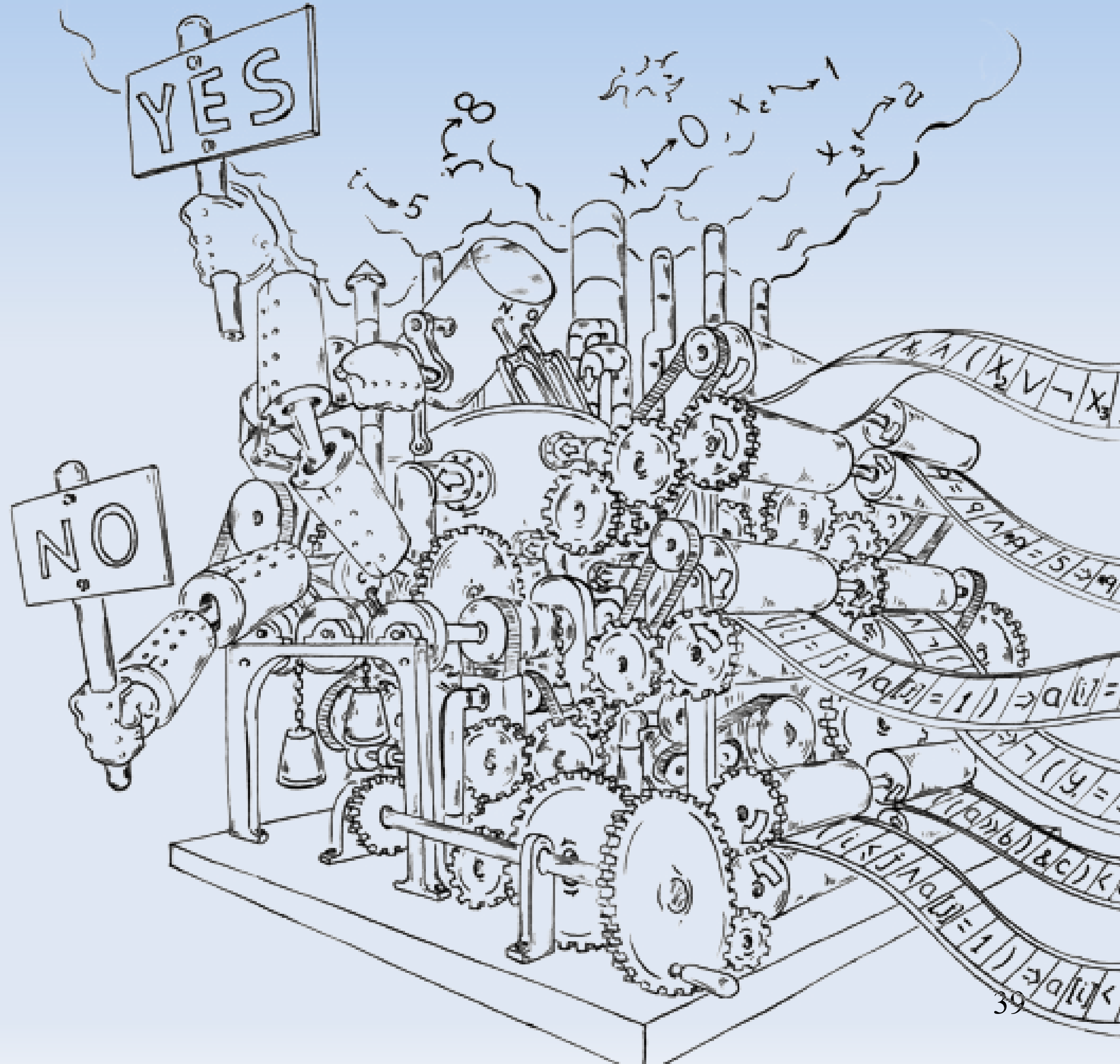
...

Do not preserve length ...

Other operations

- String reversal
- Context-free grammars
- String-to-number conversions
- Replace-all with symbolic arguments
- ...

Solving String Constraints



Bit of Solver History

- Bounded-length solvers
  - Bit-vector-based: Hampi, Kaluza
  - CP-based: Gecode
- Automata-based tools
  - Stranger, TRAU
- SMT/DPLL/CDCL-based methods
  - Z3-str/2/3, CVC4, S3/p, Norn, Sloth

(+ much theoretic work)

Solving Word Equations

- What are the solutions those equations?

$$s' = \text{`}a\text{'} \cdot s \cdot \text{`}b\text{'}$$

$$x \cdot y = u \cdot \text{`}ab\text{'} \cdot v$$

Nielsen's transformation

(also called Levi's lemma)

Theorem

$$uw = xy \quad \Leftrightarrow \quad \begin{cases} \exists t. \;\; u = xt \wedge y = tw; \text{ or} \\ \exists t. \;\; x = ut \wedge w = ty \end{cases}$$

Nielsen's transformation

(also called Levi's lemma)

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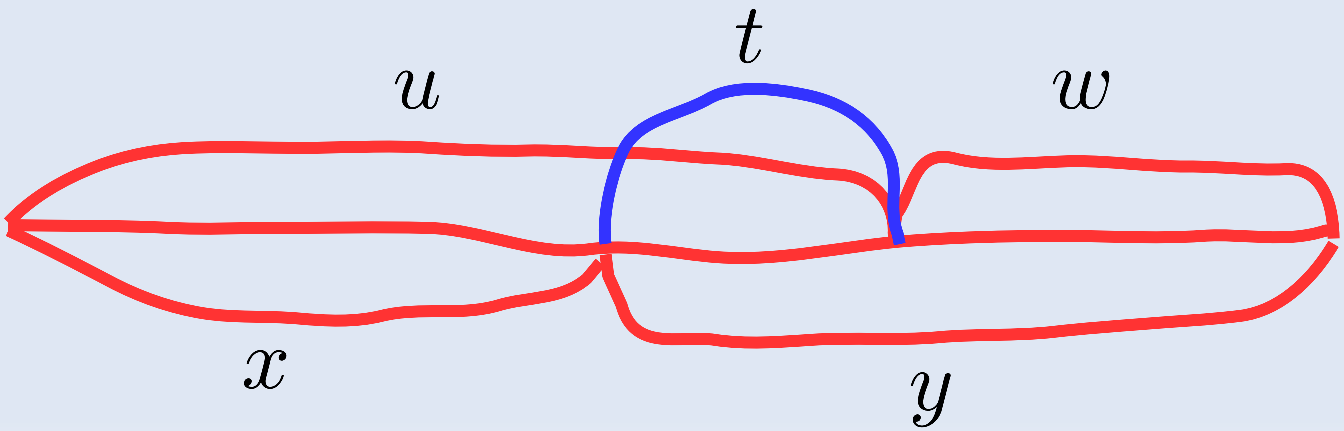

As a tableau rule

Nielsen's transformation

$$\frac{x\alpha = y\beta}{\begin{array}{c|c} x = yz & y = xz \\ z\alpha[x/yz] = \beta[x/yz] & \alpha[y/xz] = z\beta[y/xz] \end{array}}$$

$(z \text{ fresh})$

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$(z \text{ fresh})$

$$\frac{\alpha\beta = \alpha\gamma}{\beta = \gamma} \qquad \frac{\text{`}a\text{'}\alpha = \text{`}b\text{'}\beta}{*} \qquad \ldots$$

In the example

$$x \cdot y = u \cdot \text{`}ab\text{'} \cdot v$$

How about this one?

$$x \cdot y = y \cdot z$$

How about this one?

$$x \cdot y = y \cdot z$$

$$x = yt$$
$$ty = z$$

$$y = xt$$
$$xt = tz$$

How about this one?

$$x \cdot y = y \cdot z$$

$$x = yt$$
$$ty = z$$

$$y = xt$$
$$xt = tz$$

Cycle!

What can be done?

Ignore cycles and hope for the best!

Identify fragments for which NT is guaranteed to terminate

- Acyclic; straight-line

Improve NT and add termination criteria

- Makanin's method
- Simpler algorithms for **quadratic** equations

Quadratic word equations

Definition
A word equation is **quadratic** if each variable occurs at most twice in the equation.

- E.g.

$$x \cdot y = u \cdot \text{`}ab\text{'} \cdot v$$

$$x \cdot y = y \cdot z$$

- Consider satisfiability of a **single quadratic** equation

Quadratic = simpler?

Nielsen's transformation

$$\frac{x\alpha = y\beta}{\begin{array}{c|c} x = yz & y = xz \\ z\alpha[x/yz] = \beta[x/yz] & \alpha[y/xz] = z\beta[y/xz] \end{array}}$$

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Quadratic = simpler?

Nielsen's transformation

$$\frac{x\alpha = y\beta}{\begin{array}{c} x = yz \\ z\alpha[x/yz] = \beta[x/yz] \end{array} \quad \Big| \quad \begin{array}{c} y = xz \\ \alpha[y/xz] = z\beta[y/xz] \end{array}}$$

$(z \text{ fresh})$

Number of variable occurrences cannot increase!

A decision procedure

Modified Nielsen rule

$$\frac{x\alpha = y\beta}{\begin{array}{c|c|c} x \to y & x \to yz & y \to xz \\ \alpha[x/y] = \beta[x/y] & z\alpha[x/yz] = \beta[x/yz] & \alpha[y/xz] = z\beta[y/xz] \end{array}}$$

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A decision procedure

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(z fresh)

Further rules

$$\frac{\alpha\beta = \alpha\gamma}{\beta = \gamma} \qquad \frac{\text{‘}a\text{’}\alpha = \text{‘}b\text{’}\beta}{*} \qquad \frac{\begin{array}{c}\alpha = \beta \\ \vdots \\ \alpha' = \beta'\end{array}}{*}$$

(equations equal up to renaming of variables)

Example

$$x \cdot \text{`}a\text{'} \cdot y = y \cdot \text{`}b\text{'} \cdot z$$

Even more rules

One-sided Nielsen rule

$$\frac{x\alpha = \text{`}a\text{'}\beta}{\begin{array}{c|c} x \to \epsilon & x \to \text{`}a\text{'}z \\ \alpha[x/\epsilon] = \text{`}a\text{'}\beta[x/\epsilon] & z\alpha[x/\text{`}a\text{'}z] = \beta[x/\text{`}a\text{'}z] \end{array}}$$

$(z \text{ fresh})$

Decision procedure?

Soundness

- If root is satisfiable, at least one branch cannot be closed

Completeness

- If root is unsat, a closed proof exists
- Follows from termination
- Open branches → satisfying assignments

Termination

- # of variable occurrences does not increase
- Up to renaming of variables, only finitely many different equations exist

Soundness argument

- Label equations $\alpha = \beta$ in the proof with:
  - $\top$ if equation is unsat
  - $\langle o, l \rangle$ if equation is sat, has o variable occurrences, and l is length of α for the shortest solution
- Order pairs $\langle o, l \rangle$ lexicographically

Lemma

In each application of the Nielsen rule, if the parent is labelled with $p < \top$, then at least one child has label $< p$.

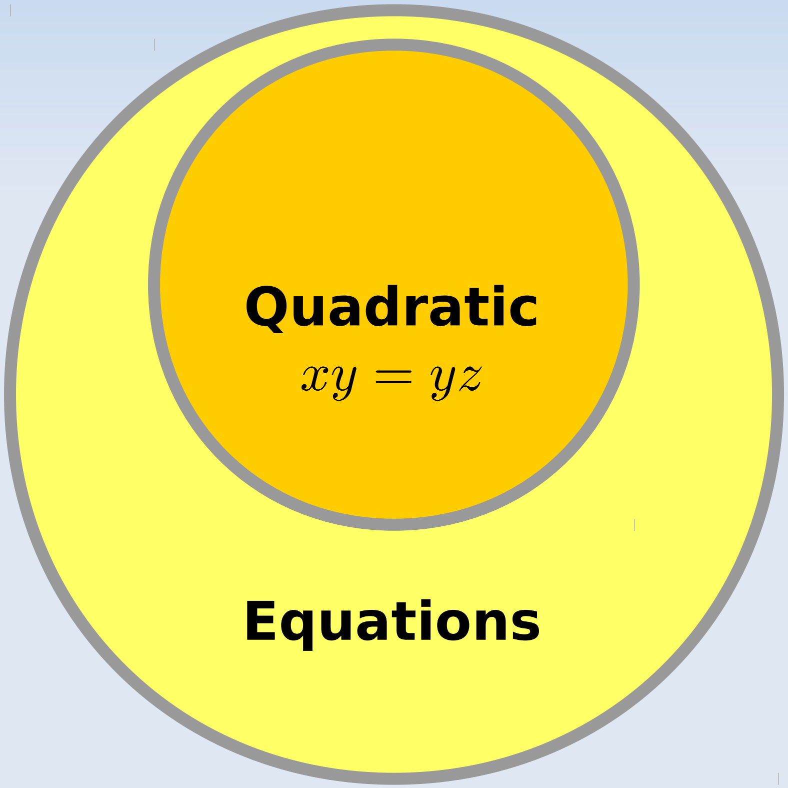
Soundness argument

- Label equations $\alpha = \beta$ in the proof with:
  - $\top$ if equation is unsat
  - $\langle o, l \rangle$ if equation is sat, has o variable occurrences, and l … r the shortest solution
- Order pairs $\langle o, l \rangle$ le…

Decreasing labels → Branch cannot be closed!

Lemma
In each application of the Nielsen rule, if the parent is labelled with $p < \top$, then at least one child has label $< p$.

Combinations ...



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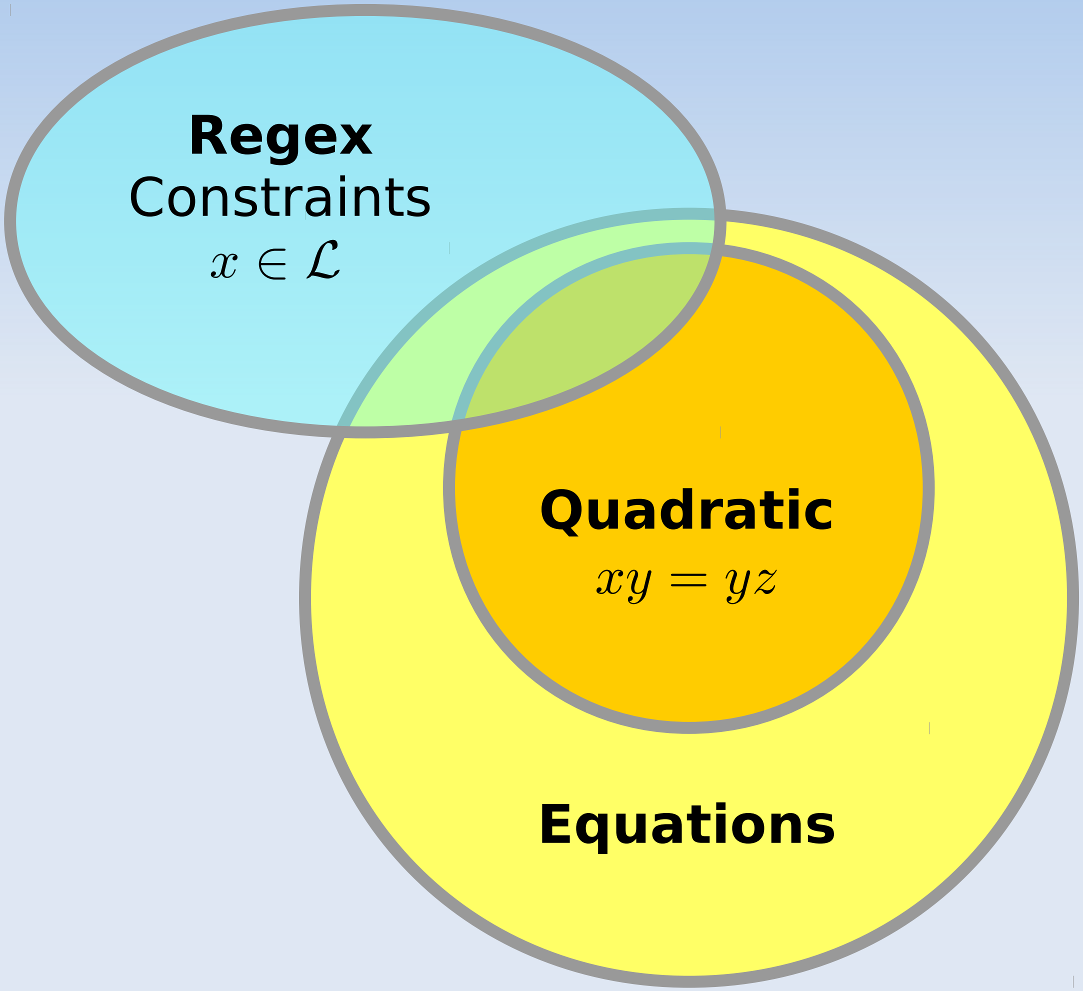

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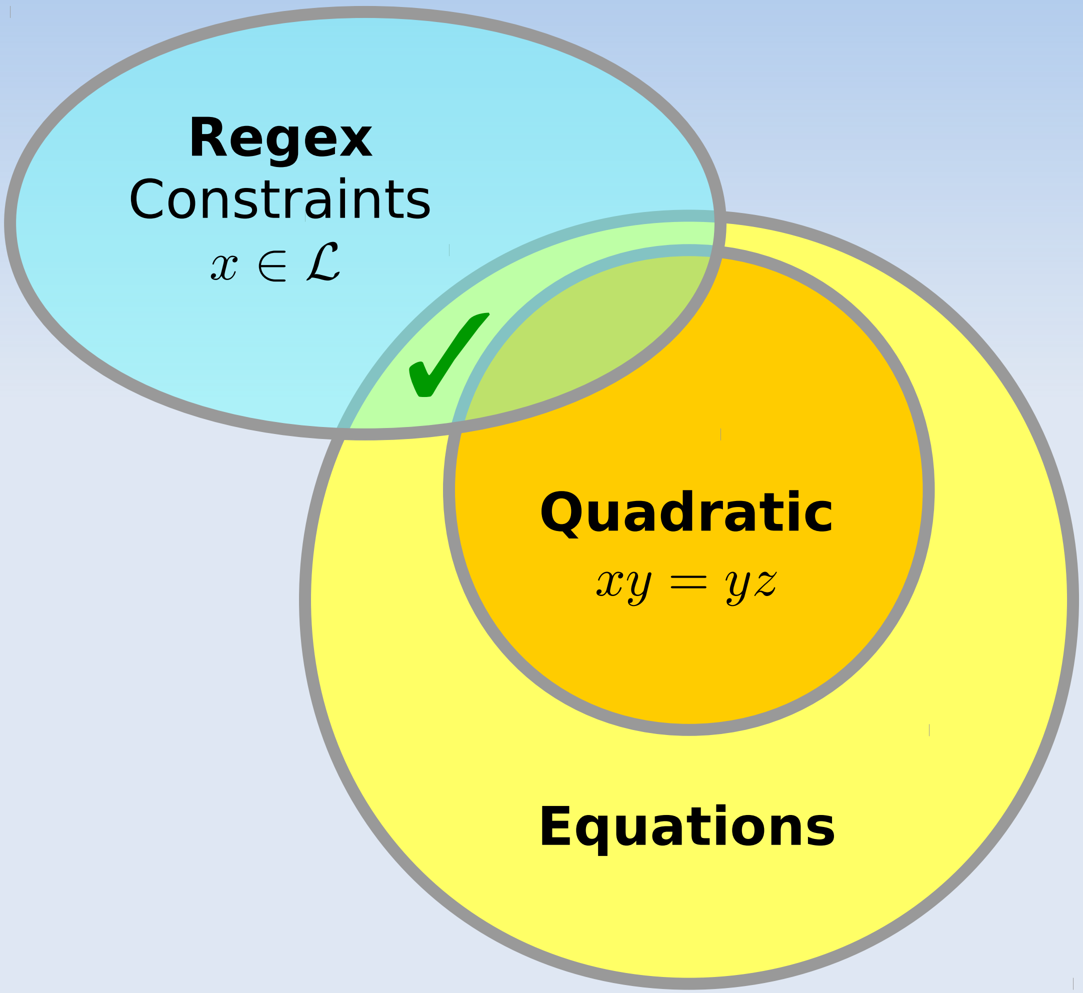

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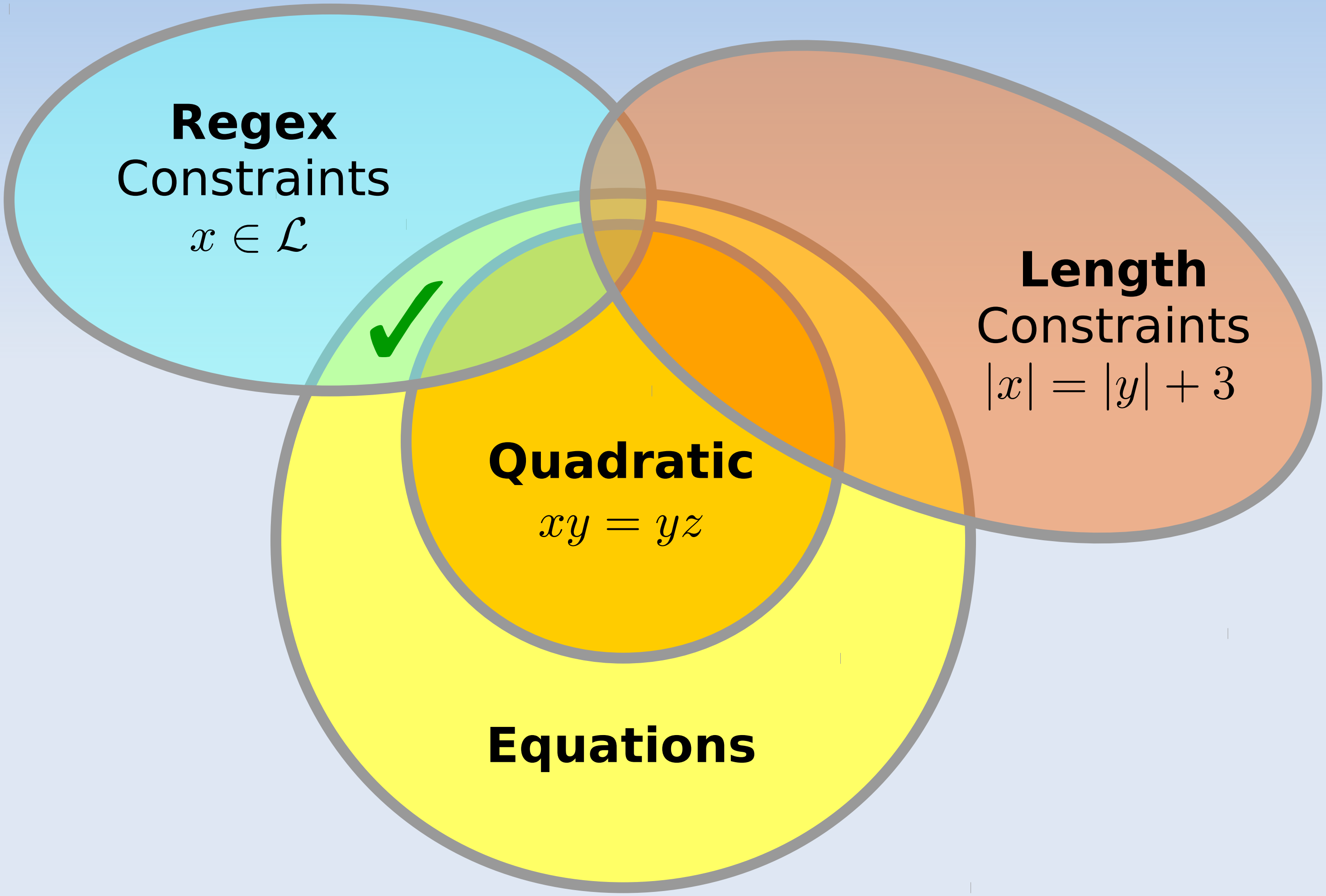

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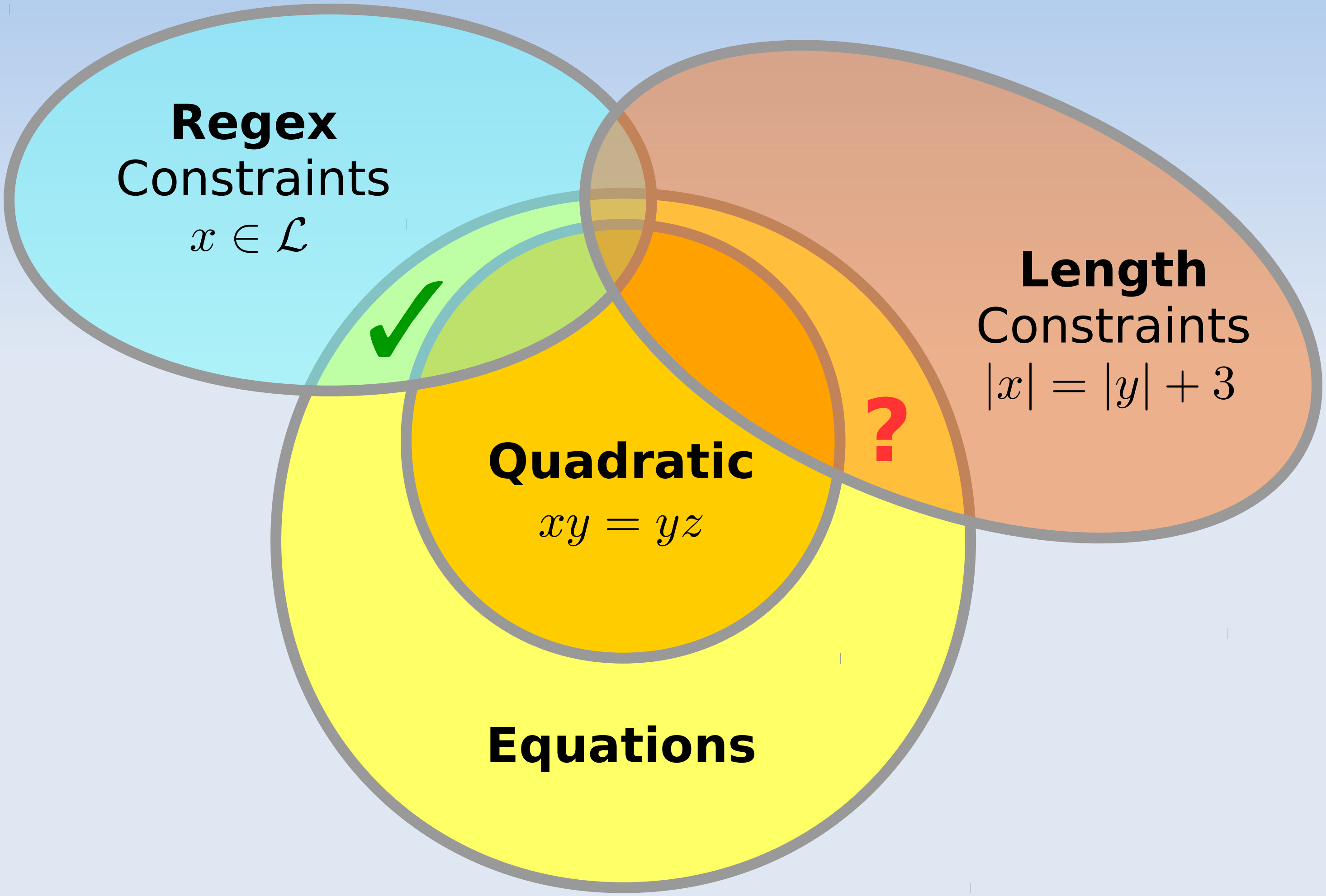

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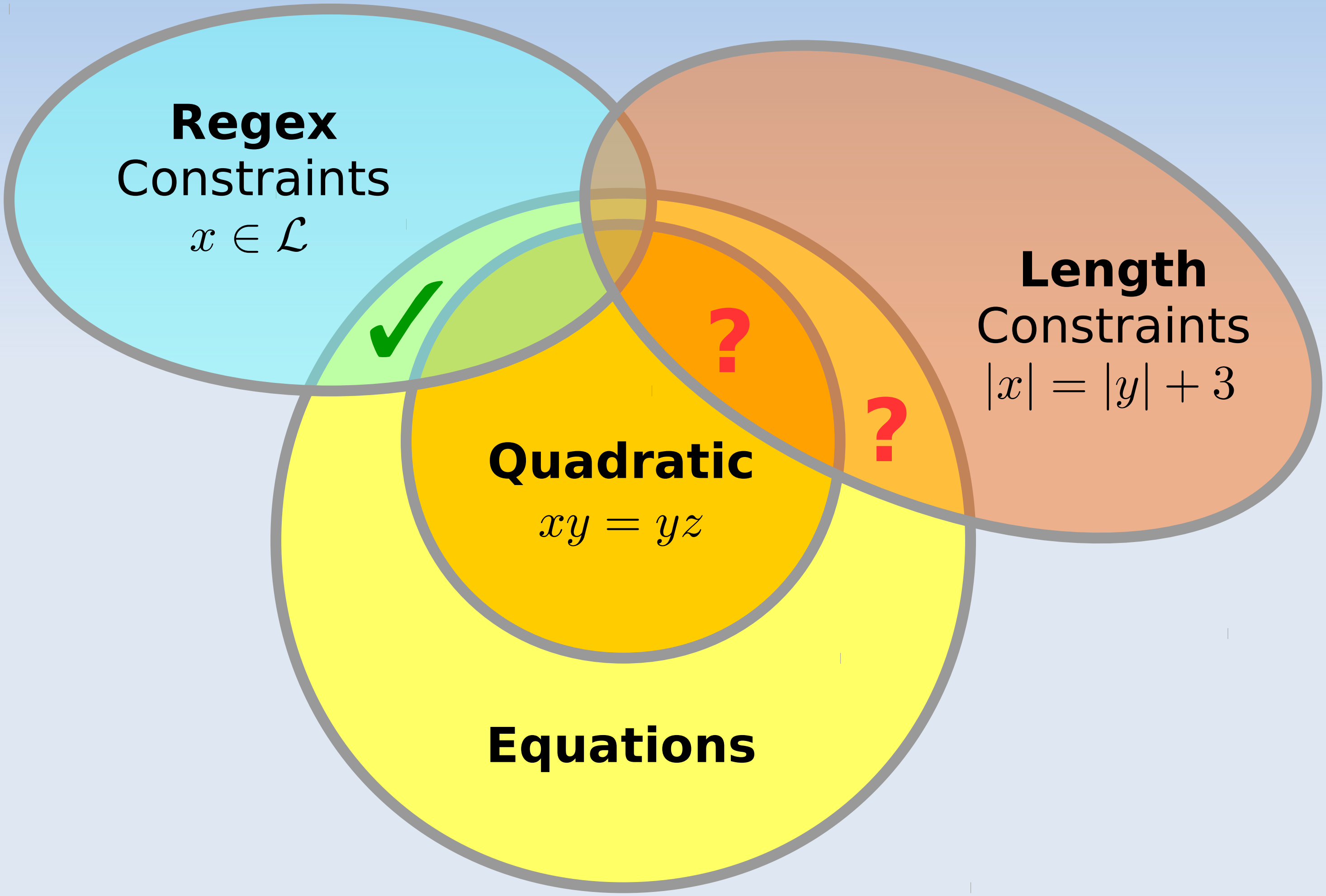

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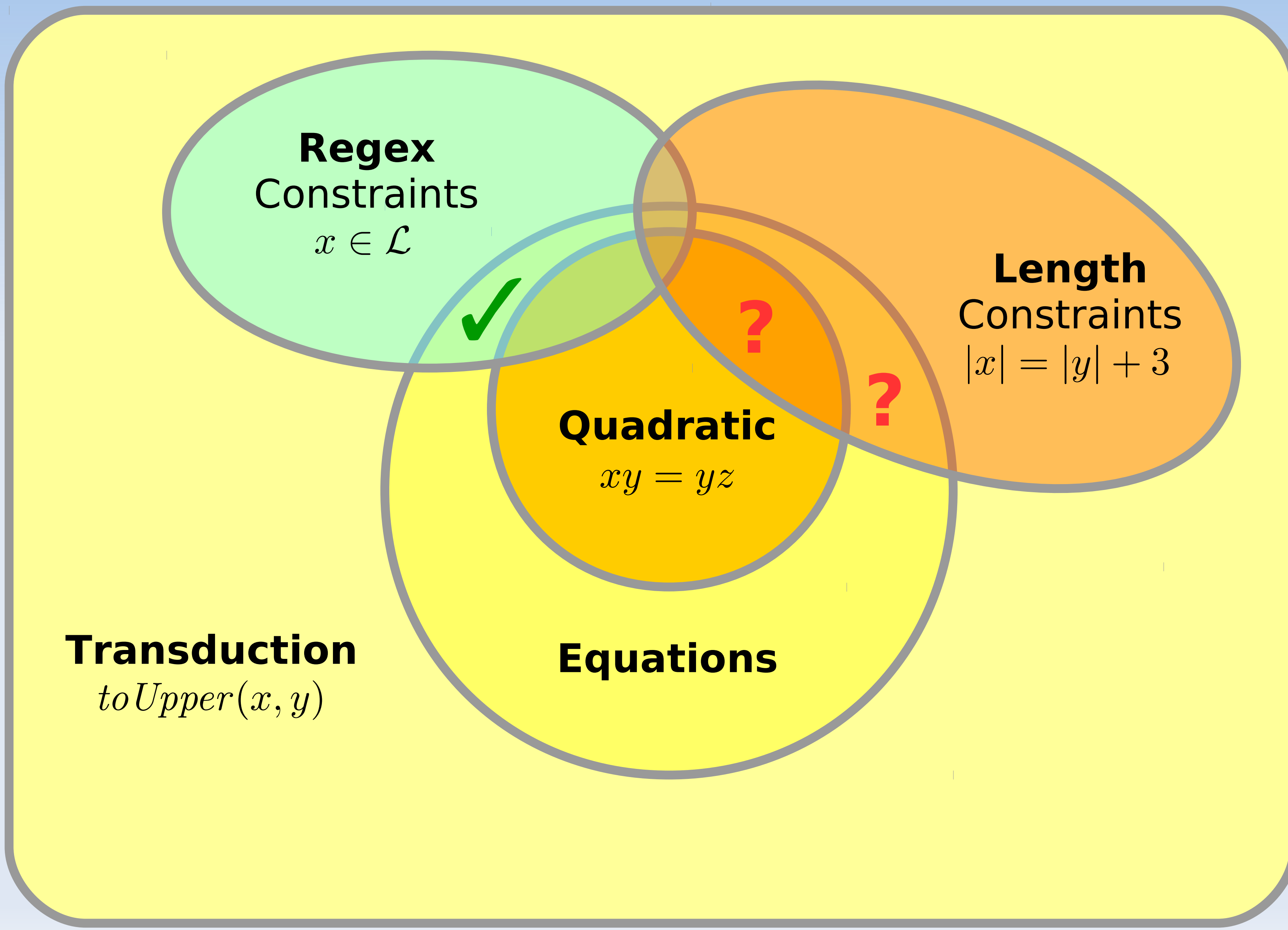

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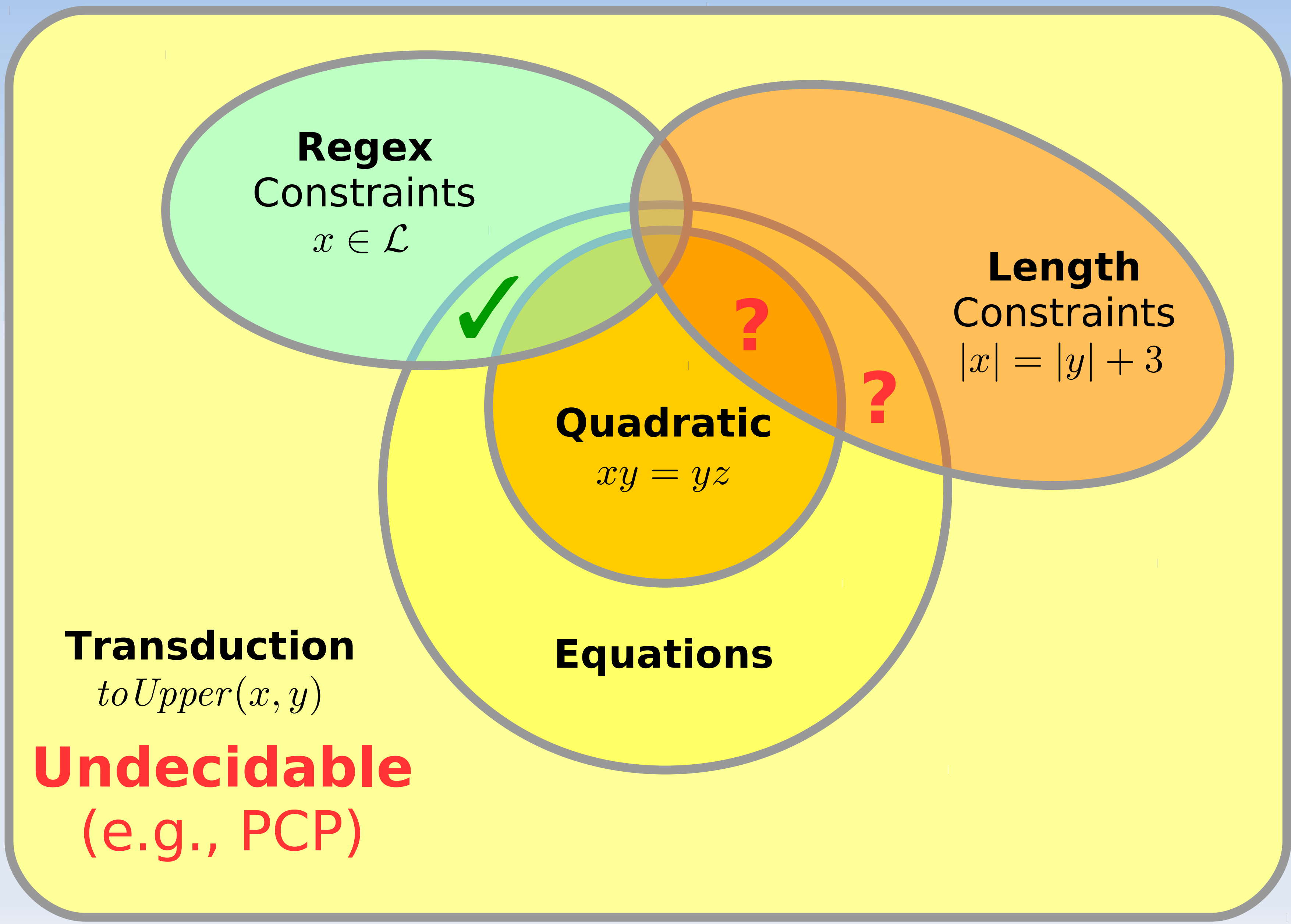

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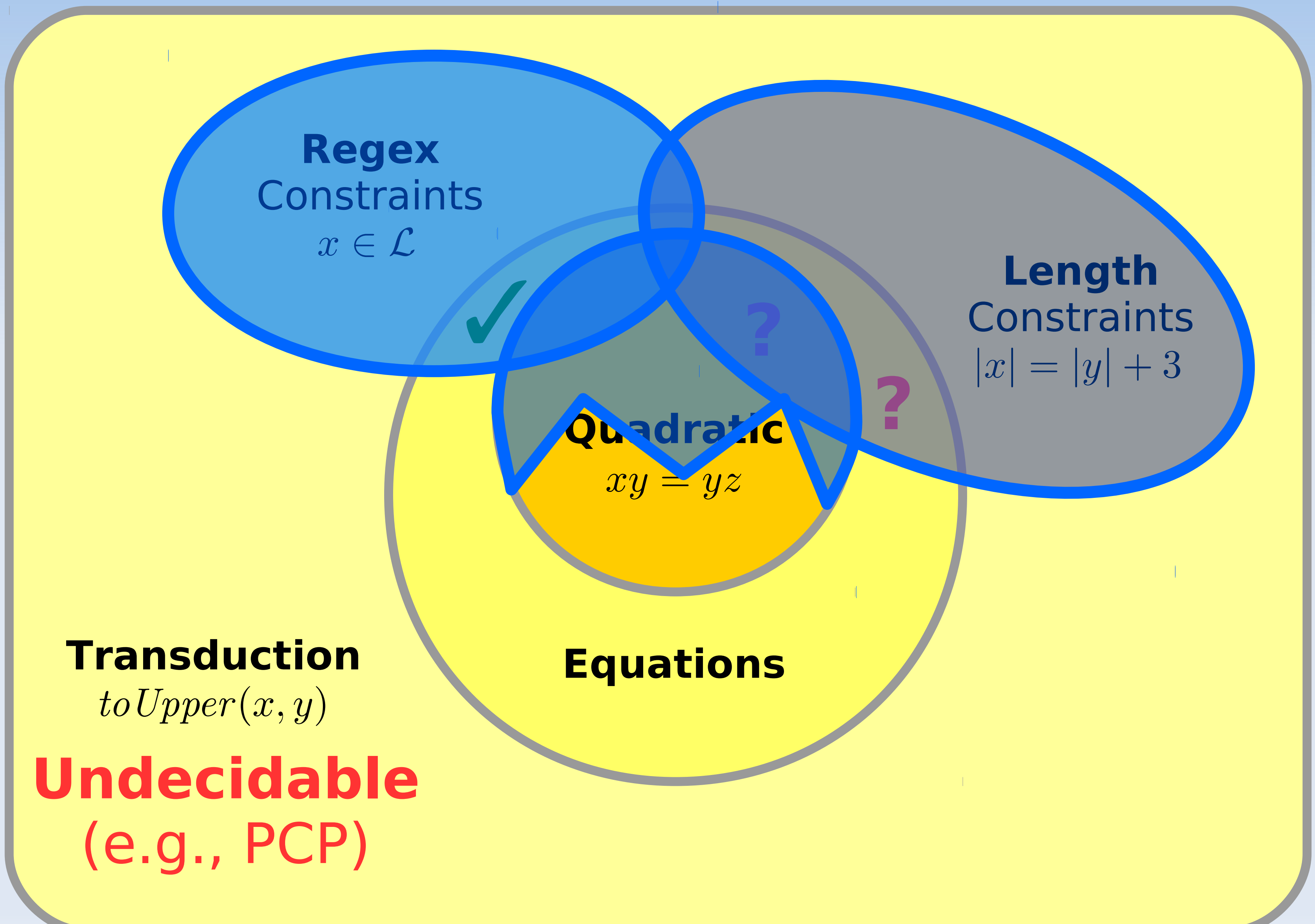

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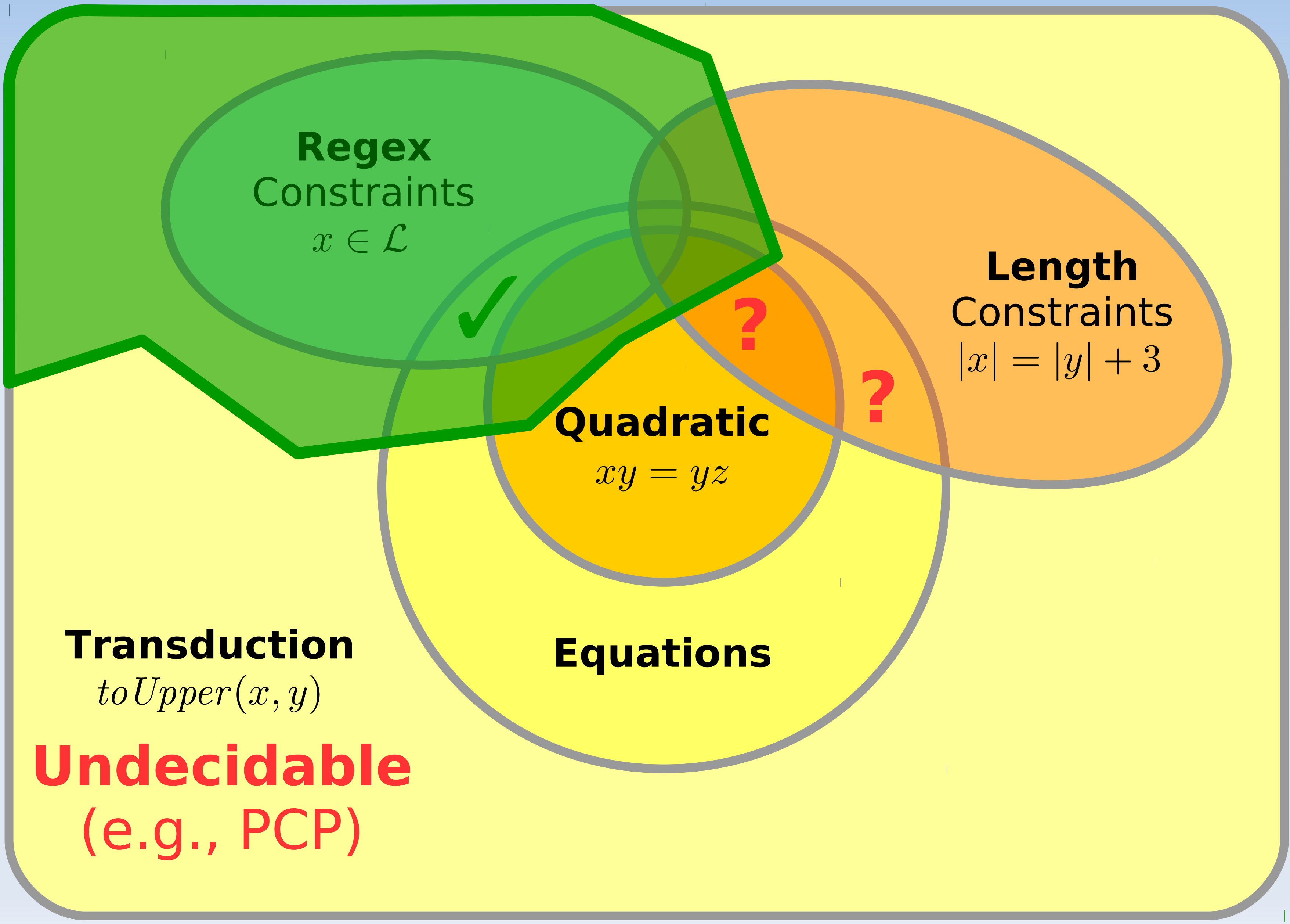

The Norn fragment

1. Boolean structure
2. Acyclic (linear) word equations
3. Regex memberships
4. Length constraints

Parosh Aziz Abdulla, Mohamed Faouzi Atig, Yu-Fang Chen, Lukás Holík, Ahmed Rezine, Philipp Rümmer, Jari Stenman: String Constraints for Verification. CAV 2014

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(a decidable fragment)

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Order in which procedure handles operators

(a decidable fragment)

Parosh Aziz Abdulla, Mohamed Faouzi Atig, Yu-Fang Chen, Lukás Holík, Ahmed Rezine, Philipp Rümmer, Jari Stenman: String Constraints for Verification. CAV 2014

Examples

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    s= 'a' + s + 'b';
}
// P3 = P2
assert(!s.contains('ba') && (s.length() % 2) == 0);
// Post = P3
```

1. Boolean structure

- Use standard DPLL/CDCL → Easy
- Just consider conjunctions of literals

- But we need to handle negation!
  - Negated word equations
  - Negated regex constraints
  - Negated length constraints

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- Just consider conjunctions of literals

- But we need to handle negation!
  - Negated word equations ?
  - Negated regex constraints ✓
  - Negated length constraints ✓

1b. Negative word eqs.

Can be reduced to positive equations:

Lemma

$$x \neq y \quad \Leftrightarrow \quad \begin{cases} \exists a \in \Sigma, u \in \Sigma^*.\ x = y \cdot a \cdot u;\ \text{or} \\ \exists a \in \Sigma, u \in \Sigma^*.\ y = a \cdot a \cdot u;\ \text{or} \\ \exists a \neq b \in \Sigma, p, u, v \in \Sigma^*. \\ \qquad x = p \cdot a \cdot u \wedge y = p \cdot b \cdot v \end{cases}$$

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Large alphabets
→ *a*, *b* need to be
handled symbolically
in practice

1b. Negative word eqs.

Can be reduced to positive equations:

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$$x \neq y \quad \Leftrightarrow \quad \begin{cases} \exists a \in \Sigma, u \in \Sigma^*.\ x = y \cdot a \cdot u; \text{ or} \\ \exists a \in \Sigma, u \in \Sigma^*.\ y = a \cdot a \cdot u; \text{ or} \\ \exists a \neq b \in \Sigma, p, u, v \in \Sigma^*. \\ \qquad x = p \cdot a \cdot u \wedge y = p \cdot b \cdot v \end{cases}$$

Theorem

Any Boolean combination of word equations can be reduced to a single word equation with the same set of solutions (when projected to the original set of variables).

2. Acyclic word equations

- Reduce to solved form by systematic application of Nielsen's transformation:

$$x_1 = t_1 \wedge \cdots \wedge x_n = t_n$$

($x_1, \ldots, x_n$ do not occur in $t_1, \ldots, t_n$)

- After that, eliminate equations by inlining!

3. Regular expressions

- Membership tests with **concatenation** can be split:

$$s \cdot t \in \mathcal{L} \quad \rightsquigarrow \quad \bigvee_{i=1}^{n} s \in \mathcal{L}_1^i \wedge t \in \mathcal{L}_2^i$$

- Tests with **same left-hand side** can be merged:

$$x \in \mathcal{L}_1 \wedge x \in \mathcal{L}_2 \quad \rightsquigarrow \quad x \in \mathcal{L}_1 \cap \mathcal{L}_2$$

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Disjunction over states of automaton representing $\mathcal{L}$

- Tests with **same left-h** be merged:

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4. Length constraints

- Compute the **length abstraction** of each regex constraint:

$$x \in \mathcal{L} \quad \rightsquigarrow \quad |x| \in \{|w| \mid w \in \mathcal{L}\}$$

- Conjoin length abstractions with other length constraints and check **satisfiability**

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A Presburger formula that can be extracted in linear time from $\mathcal{L}$

5. Optimisations ...

- E.g., exploit length information when splitting equations or regexes

(still too slow ...)

The Sloth fragments

1. Boolean structure (no negation)
2. Straight-line word equations
3. *n*-track transducer constraints

Lukás Holík, Petr Janku, Anthony W. Lin, Philipp Rümmer, Tomás Vojnar: String constraints with concatenation and transducers solved efficiently. PACMPL 2(POPL): 4:1-4:32 (2018)

The Sloth fragments

1. Boolean structure (no negation)
2. Straight-line word equations
3. *n*-track transducer constraints

→ also decidable!

Lukás Holík, Petr Janku, Anthony W. Lin, Philipp Rümmer, Tomás Vojnar: String constraints with concatenation and transducers solved efficiently. PACMPL 2(POPL): 4:1-4:32 (2018)

Example

Conclusions:
Are we there yet?

Expressiveness

Efficiency

Precision/
guarantees

Joint work with ...

- Parosh Aziz Abdulla
- Mohamed Faouzi Atig
- Yu-Fang Chen
- Bui Phi Diep
- Lukás Holík
- Petr Janků
- Anthony W. Lin
- Ahmed Rezine
- Jari Stenman
- Tomás Vojnar
- *and others*

Further topics

- The SMT-LIB standard for strings (work in progress ...)
- Solver applying under- and over-approximations
- Context-free grammars
- Model counting
- ...